Kuwait University | Math 102 | January 14, 2008
Dept of Math & Comp Sci | Final Examination | Duration: 2 hours

Turn off your cell phone. The use of any type of calculator or graphing utility is prohibited. Answer the following questions.

1. Let $f(x) = \int_2^x \frac{1}{t} e^{-t} dt, \quad x > 0.$

   (a) Show that f is one-to-one on its domain. (2 points)

   (b) Explain why the point $P(0, 2)$ is on the graph of f^{-1}. (1 point)

   (c) Find the slope of the tangent line to the graph of f^{-1} at $P(0, 2)$. (2 points)

2. (a) Prove the identity (3 points)

$$2\tan^{-1} e^x - \tan^{-1}(\sinh x) = \frac{\pi}{2}, \qquad -\infty < x < \infty.$$

   (b) Find the limit $\lim_{x \to 0} \left[\exp(\frac{1}{x^2})\right]^{(\cosh x - 1)}$ if it exists. (4 points)

3. Evaluate the following integrals. (4 points each)

   (a) $\int \frac{e^{-2x^{-2}}}{x^5} dx$ (b) $\int \frac{\cot x}{4\sin^2 x - 1} dx.$

4. Find the value of the improper integral $\int_1^\infty \frac{1}{x\sqrt{x^2+4}} dx$ if convergent. (4 points)

5. Find the centroid of the region R, which is bounded by the curve $y = (x+1)^{2007}$, the x-axis and the lines $x = -1$ and $x = 0$. (4 points)

6. Let C be the parametric curve

$$x = \cos^3 t, \qquad y = \sin^3 t, \qquad 0 \le t \le \frac{\pi}{2}.$$

   (a) Find the point on C where the tangent line is parallel to the line $y = -\sqrt{3}x$. (4 points)

   (b) Find the length of C. (4 points)

7. Find the area of the region which lies inside the circle $r = \frac{1}{2}$ and outside the cardioid $r = 1 - \cos\theta$. (4 points)